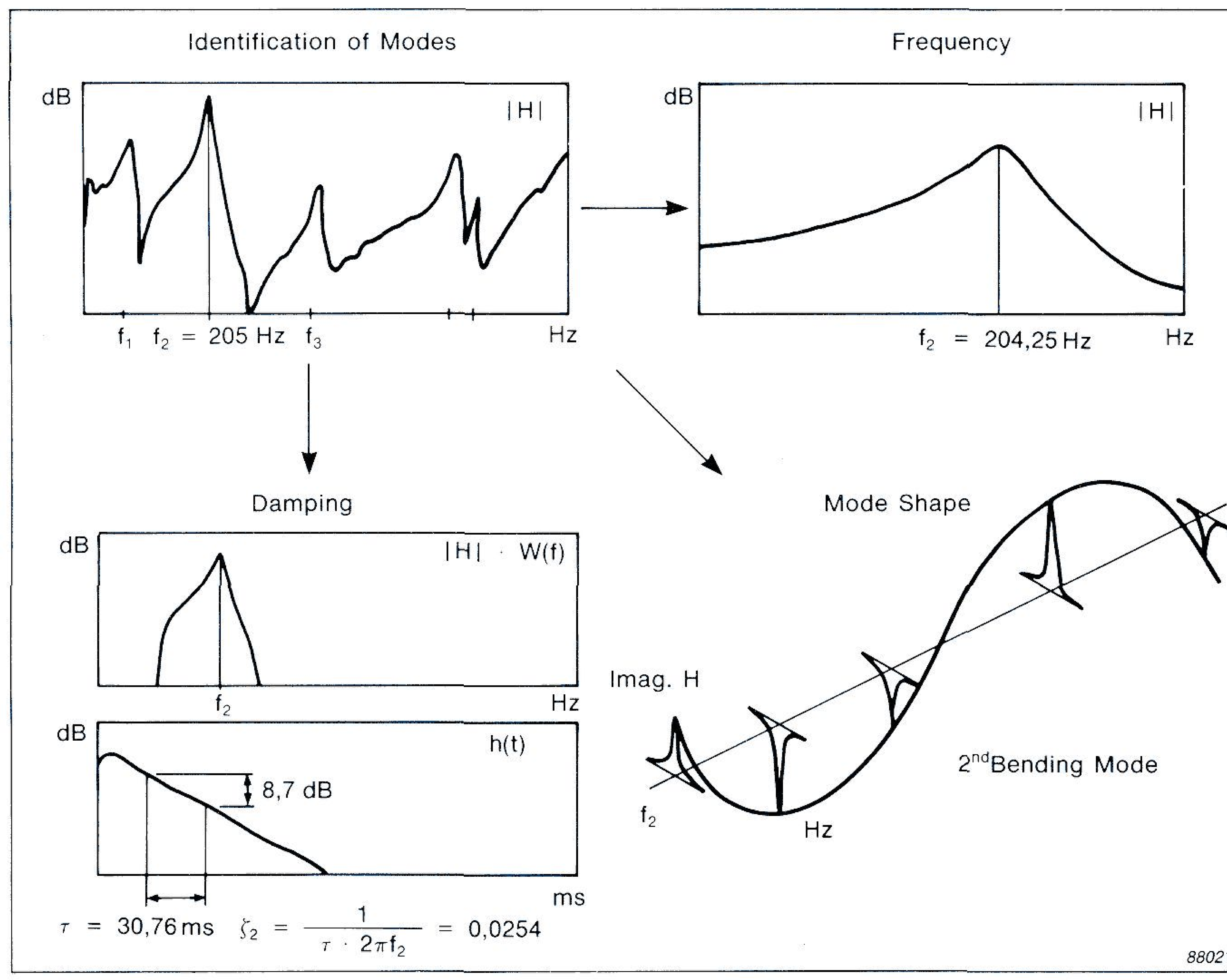
How to determine the modal parameters of simple structures

The modal parameters of simple structures can be simply established with the aid of a Dual-Channel Signal Analyzer Type 2032 or 2034. This application note describes how to measure the modal frequencies by inspection of frequency response functions, how to determine the modal damping with the aid of the frequency weighting function included in the analyzer, and how to establish the mode shapes by examining the value of the imaginary part of the frequency response function.



The frequency response function of a structure can be separated into a set of individual modes. By using a Dual-Channel Signal Analyzer Type 2032 or 2034, each mode can be identified in terms of frequency, damping and mode shape.

Introduction

In practice, nearly all vibration problems are related to structural weaknesses associated with the resonant behaviour excited by operational forces. However, the complete dynamic behaviour can be viewed as a set of individual modes of vibration, each having a characteristic resonance frequency, damping, and mode shape. By using these modal parameters to model the structure, problems at specific resonances can be examined and then solved.

The first stage in modelling the dynamic behaviour of a structure is to determine the modal parameters as follows:

- The resonance, or modal, frequency.
- The damping at resonance – the modal damping.
- The mode shape.

The modal parameters can be determined from a set of frequency response measurements between a reference point and a number of measurement points on the structure. The modal frequencies and dampings can be found from all frequency response measurements on the structure (except those for which the excitation or response measurement is in a nodal position, that is, where the displacement is zero). However, to accurately modal the mode shape, frequency response measurements must be made over a number of points completely covering the structure.

In practice, these types of frequency response measurements are made easy by using a Dual-Channel Signal Analyzer such as the Type 2032 or Type 2034. The excitation force (from either an impact hammer or a vibration exciter provided with a pseudo-random noise signal) is measured by a force transducer, and the resulting signal is supplied to the Channel A input. The response is measured by an accelerometer, and the resulting signal is supplied to the Channel B input. Consequently, the frequency response represents the structure's accelerance since it is the complex ratio of the acceleration to force, in the frequency domain. For impact hammer excitation, the response position is fixed and is used as the reference position, and the hammer is moved around and used to excite the structure at every point corresponding to a point in the model. For vibration exciter excitation, the excitation point is fixed and is used as the reference position, while the response accelerometer is moved around the structure. A typical instrumentation set-up is illustrated in Figure 1.

Simple structures

Structures which exhibit lightly coupled modes are usually referred to as *simple structures*. The modes are not closely spaced, and are not heavily damped (see Figure 2). At resonance, a simple structure behaves predominantly as a single-degree-of-freedom system, and the modal parameters can be determined relatively easily with a Brüel & Kjær Dual-Channel Signal Analyzer.

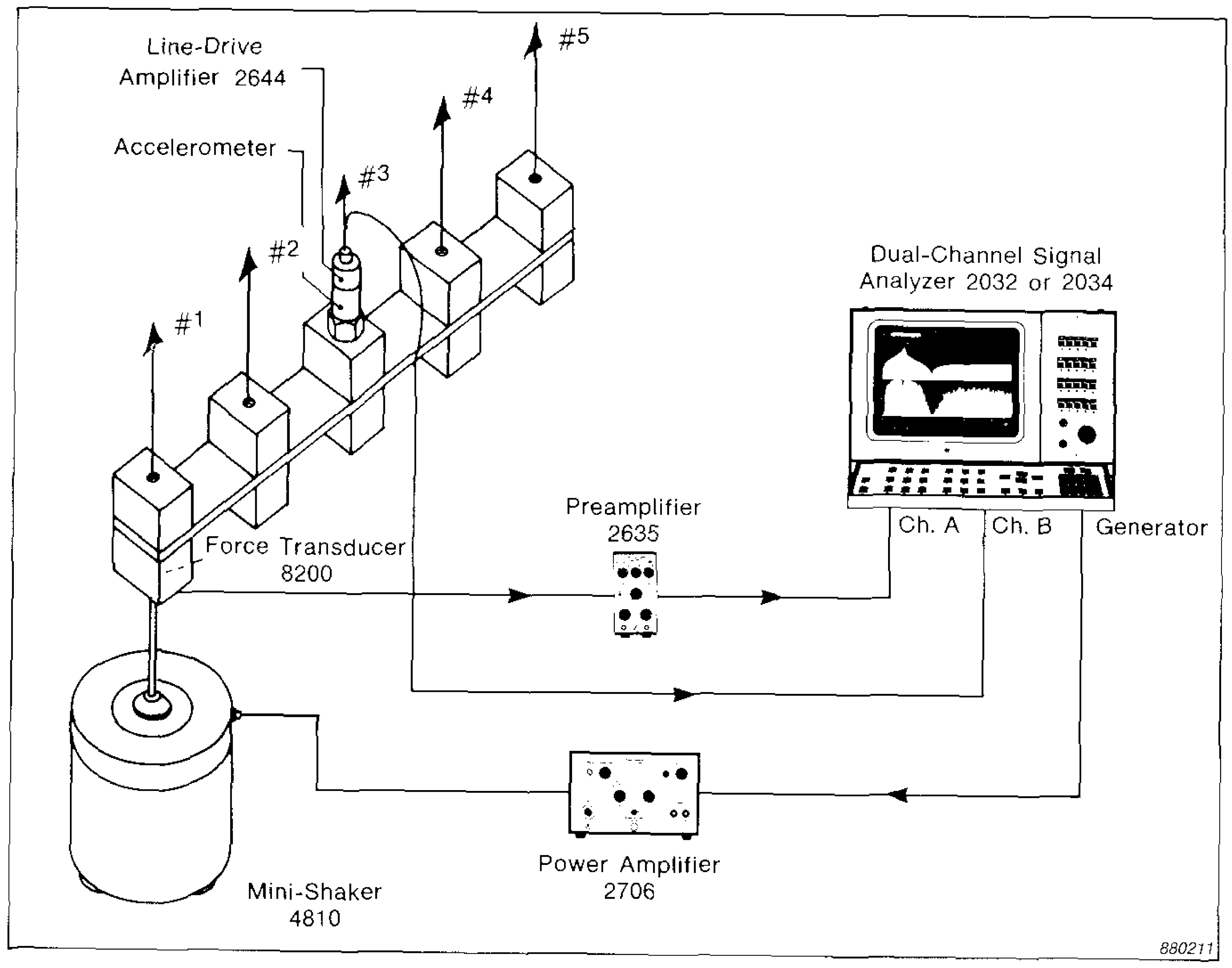


Fig. 1. An instrumentation set-up, using broadband pseudo-random force excitation provided by a vibration exciter.

Determination of the modal frequencies

The resonance frequency is the easiest modal parameter to determine. A resonance is identified as a peak in the magnitude of the frequency response function, and the frequency f_r at which it occurs is found using the analyzer's cursor as shown in Figure 3.

The frequency resolution of the analysis determines the accuracy of the frequency measurement. Greater accuracy can be attained by reducing the frequency range of the baseband measurement, for resonances at low frequencies, or making a zoom measurement, as shown in Figure 4.

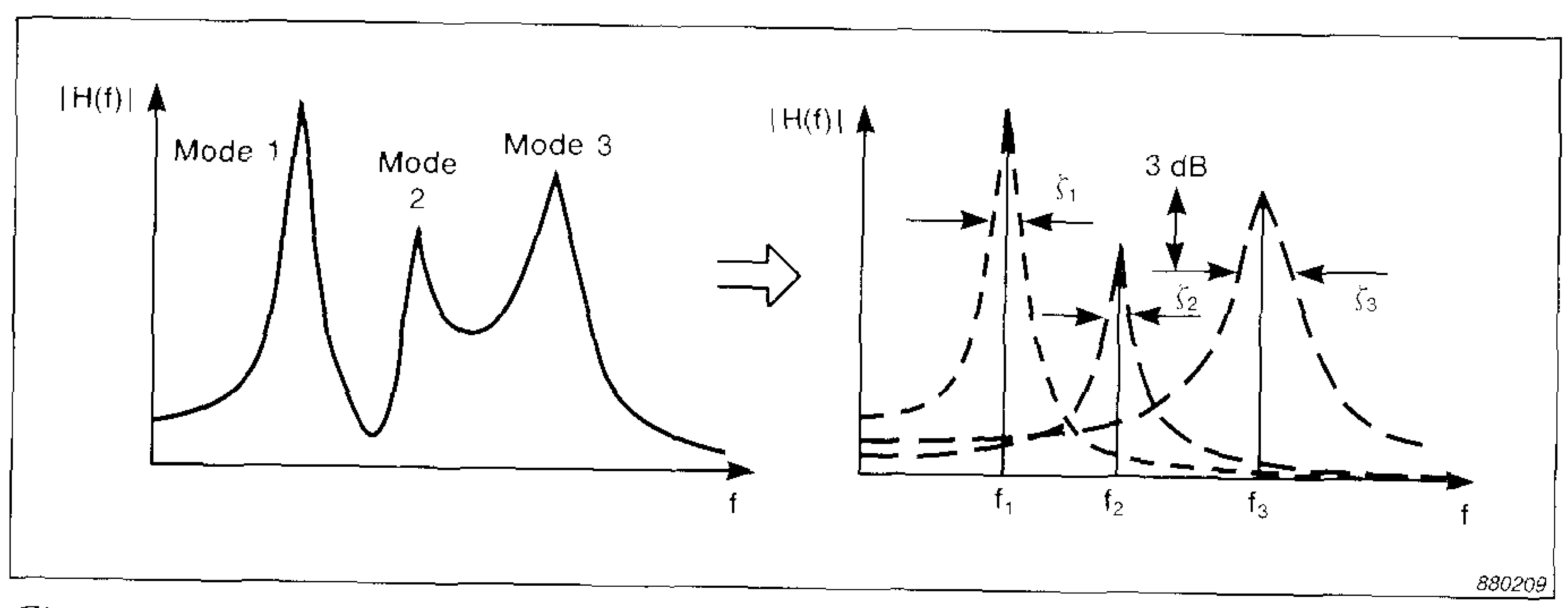


Fig. 2. The frequency response of simple structures can be split up into individual modes, each mode behaving as a single-degree-of-freedom system.

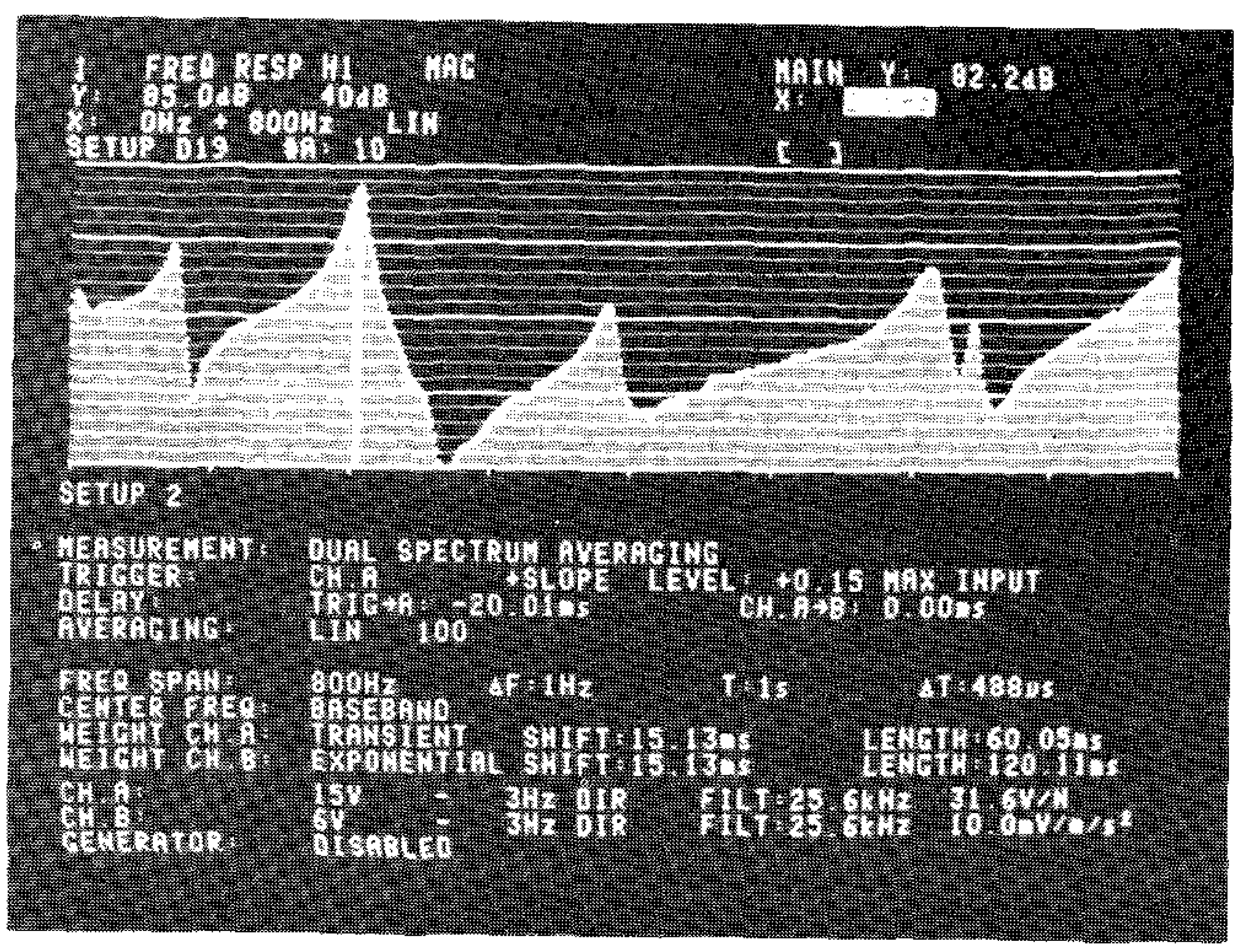


Fig. 3. Magnitude of the frequency response function for the test structure.

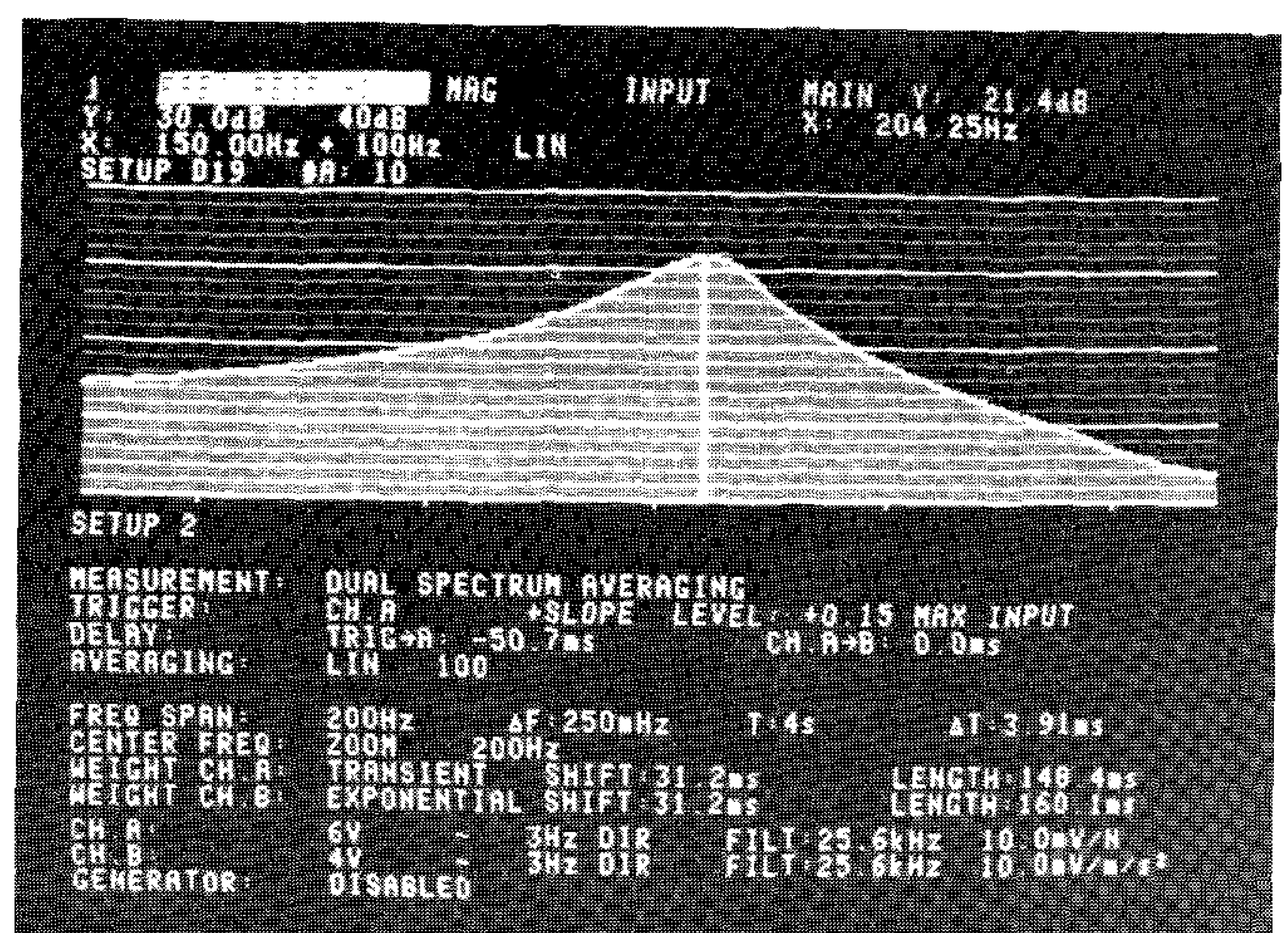


Fig. 4. Zoom measurement for achieving greater frequency resolution in the determination of the resonance frequency for the second mode.

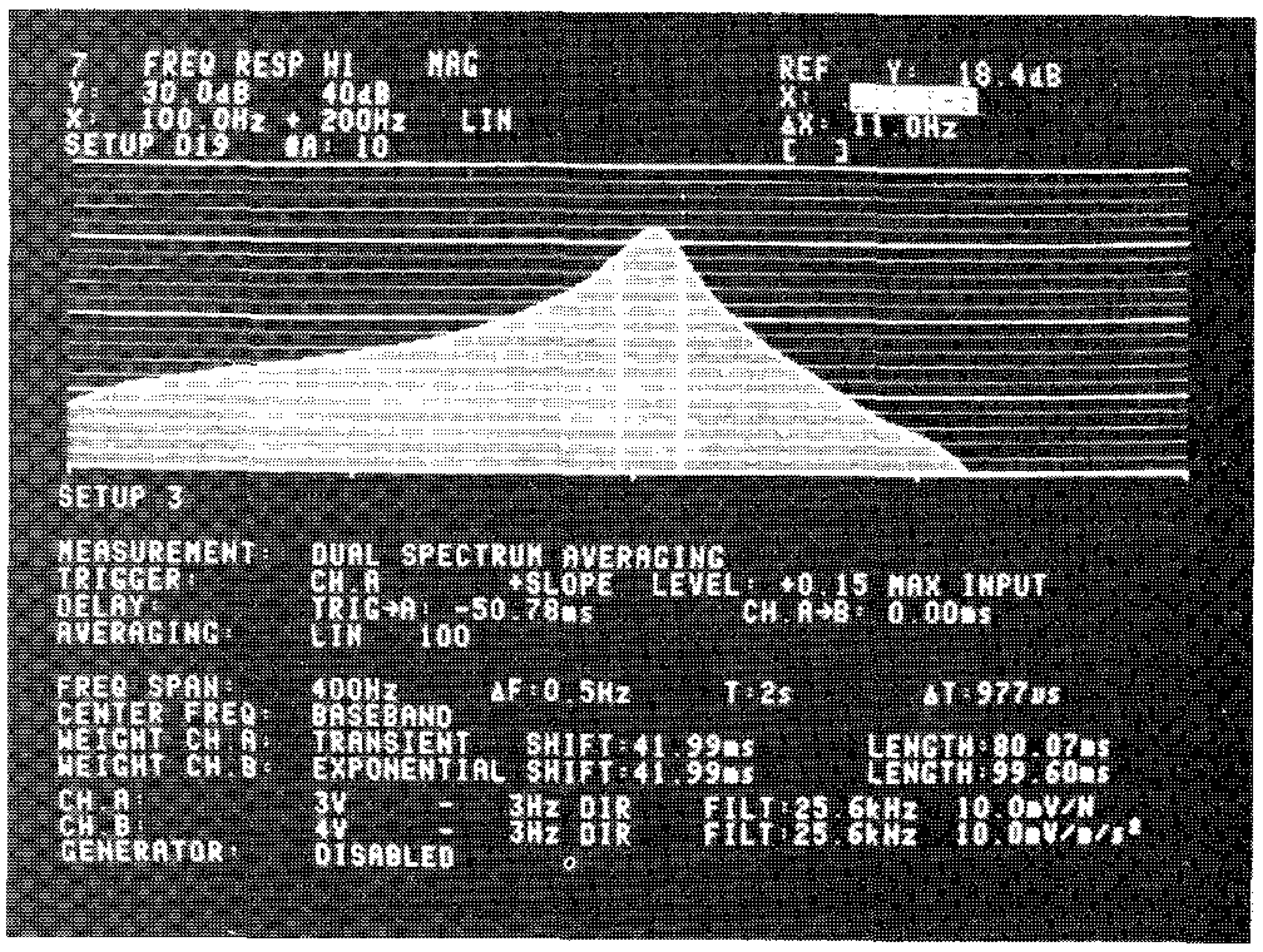


Fig. 5. Determination of the damping for the second mode by indentification of the half-power points.

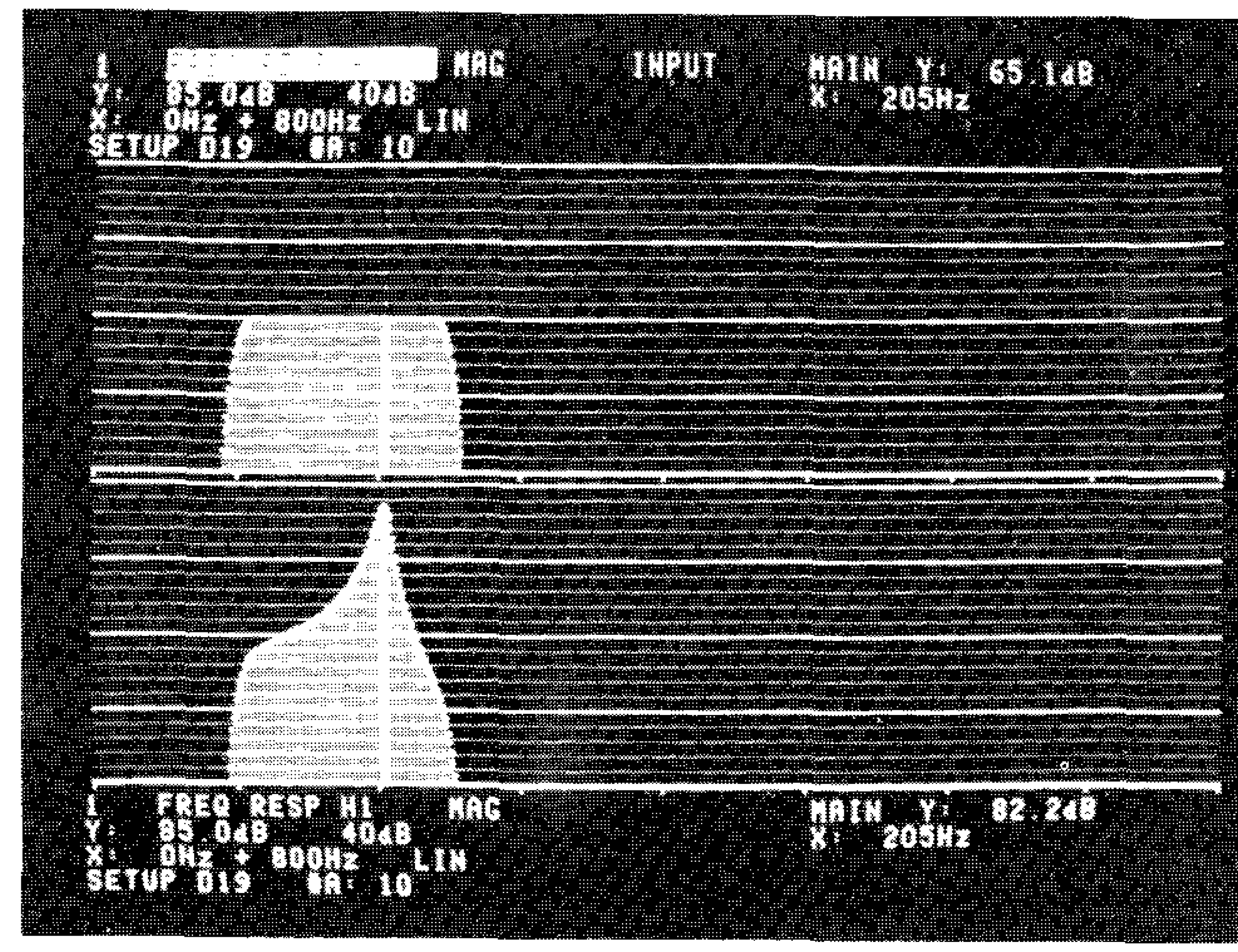


Fig. 6. The frequency weighting function of the Type 2032 and 2034 allows a single mode to be isolated from the frequency response function.

Determination of the modal damping

The classical method of determining the damping at a resonance, using a frequency analyzer, is to identify the half power (–3 dB) points of the magnitude of the frequency response function (see Figure 5). For a particular mode, the damping ratio ζ_r can be found from the following equation:

$$\zeta_r = \frac{\Delta f}{2 f_r}$$

where Δf is the frequency bandwidth between the two half power points. The use of the analyzer's reference cursor greatly simplifies the determination of the bandwidth Δf.

The accuracy of this method is dependent on the frequency resolution used for the measurement because this determines how accurately the peak magnitude can be measured. For lightly damped structures, high resolution analysis is required to measure the peak accurately, consequently, a zoom measurement at each resonance frequency is normally required to achieve sufficient accuracy. This means that a new measurement must be made for each resonance.

However, the Dual-Channel Signal Analyzers Type 2032 and 2034 can be used to determine the damping ratio by an alternative method which requires no new measurements. By using a frequency weighting function (window) to isolate a single mode from the frequency response function (see Figure 6), the impulse response function for that mode alone is easily produced.

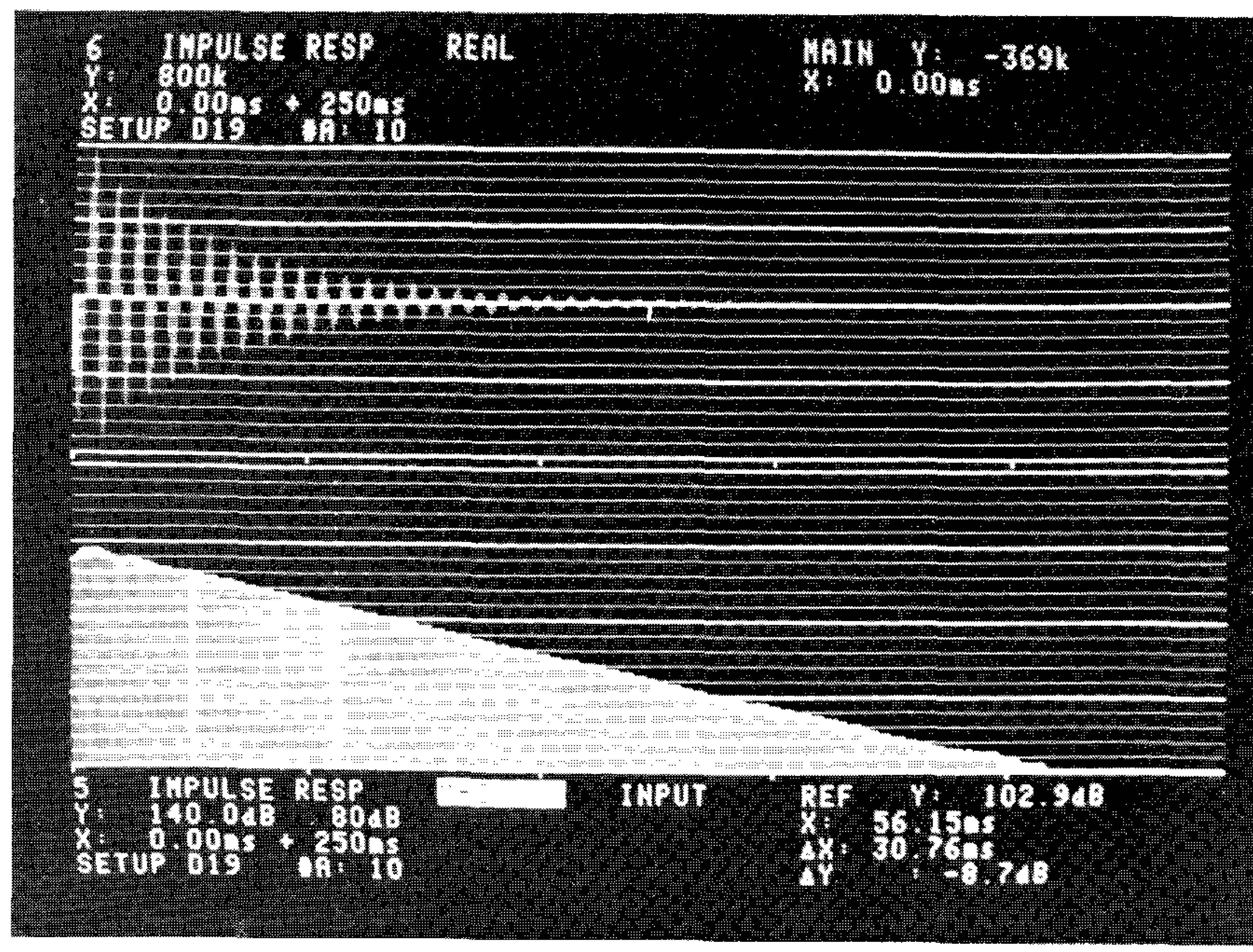


Fig. 7. The impulse response function of the isolated mode can be represented by its magnitude and displayed on a logarithmic scale to enable easy calculation of the decay rate.

The magnitude (envelope) of the impulse response function can then be displayed by virtue of the Hilbert transform facility incorporated in the analyzers.

Since a simple structure behaves as a single-degree-of-freedom system at each resonance, the impulse response function of the windowed resonance will show characteristic exponential decay τ. By displaying the magnitude on a logarithmic scale the impulse response is represented by a straight line (see Figure 7).

The decay rate σ_r for the isolated mode is related to the time constant τ_r by:

$$\tau_r = \frac{1}{\sigma_r}$$

The decay corresponding to time constant τ_r is given by the factor e^{-1}, or in dB: $-20\log(e) = -8{,}7\,dB$.

The damping ratio is related to the decay rate by:

$$\zeta_r = \frac{\sigma_r}{2\pi f_r} = \frac{1}{\tau_r \cdot 2\pi f_r}$$

By moving the frequency window through the frequency response function, and looking at each impulse response function in turn, the modal damping at each resonance can be determined from a single baseband measurement.

The decay rate, calculated from a frequency response function found by using hammer excitation, will be modified by the effective damping of the exponential weighting function applied to the response channel. This weighting function was added to curtail the structural ringing excited by the hammer impact. However, this error can be compensated for, easily, by using the following correction:

$$\sigma_r = \frac{1}{\tau_r} - \frac{1}{\tau_w}$$

where τ_w is the time constant of the exponential weighting function (see Figure 5 – measurement set-up). Notice that correction is unnecessary for a vibration exciter providing pseudo-random excitation.

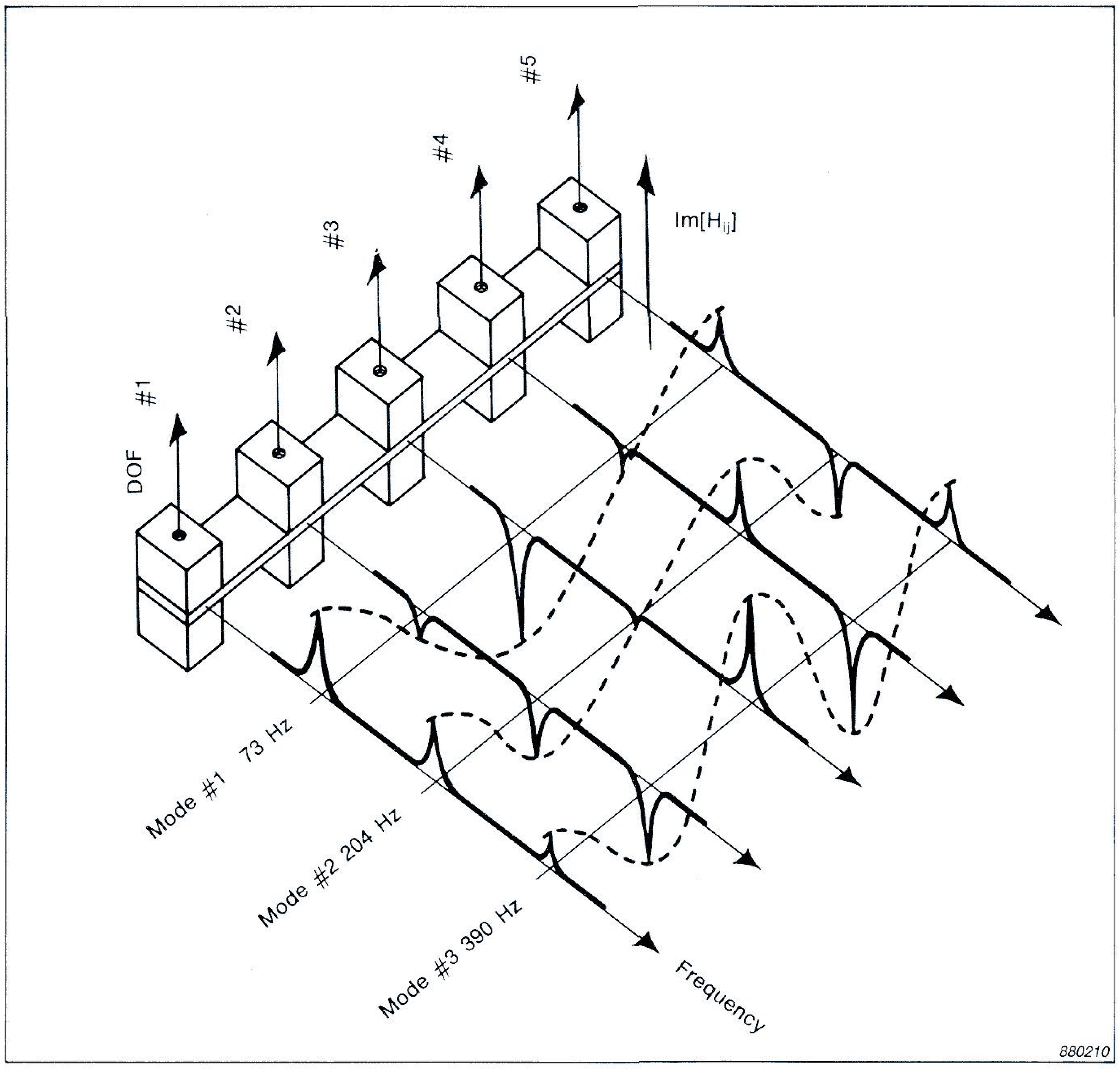


Fig. 8. The first three modes of vibration for the test structure. The modal displacements are found from the imaginary part of the frequency response function.

Determination of the mode shape

The simplest way of determining the mode shape for a structure is to use a technique called **quadrature picking**. Quadrature picking is based on the assumption that the coupling between the modes is light. In practice, mechanical structures are often very lightly damped (<1%). This implies that the modes are lightly coupled. At any frequency, the magnitude of the frequency response function is the sum of the contributions (at the particular frequency) from all modes. When there is little modal coupling between the modes, the structural response at a modal frequency is completely controlled by that mode, and so quadrature picking can be used to unravel the mode shapes.

For single degree-of-freedom systems, the frequency response function (accelerance) at resonances is purely imaginary. As a result, the value of the imaginary part of the frequency response function, at resonance for structures with lightly coupled modes, is proportional to the modal displacement. Consequently, by examining the magnitude of the imaginary part of the frequency response function at a number of points on the structure, the relative modal displacement at each point can be found. From these displacements, the mode shapes can be established. The procedure can then be repeated to determine all the required mode shapes. By making an excitation and response measurement at the same point and in the same direction, the mode shape can be scaled in absolute units.

Conclusions

The Dual-Channel Signal Analyzers Type 2032 and 2034 are ideal instruments for enabling the engineer to determine the modal parameters of simple structures. The modal frequencies are determined from the frequency response function. The modal dampings are found from the decay rate, measured from the magnitude of the impulse response function, which is produced by isolating a single mode from the frequency response function by using a frequency weighting function. To obtain the mode shape, a technique called quadrature picking is used to evaluate the modal displacements at each point of interest. The modal displacements are found from the imaginary part of the frequency response function.

By adding a computer plus the necessary software to the system, the dynamic response due to excitation forces can be simulated. Furthermore, if the vibration characteristics of prototypes tested using the system are unsatisfactory, then the influences of actual mass, stiffness and damping modifications can also be simulated. In this way, a Dual-Channel Signal Analyzer Type 2032- or 2034-based system can be expanded into a fully documented modal analysis system.

Brüel & Kjær

WORLD HEADQUARTERS: DK-2850 Nærum · Denmark · Telephone: +45 42 80 05 00 · Telex: 37316 bruka dk · Fax: +45 42 80 14 05

Australia (02) 450-2066 · Austria 02235/7550*0 · Belgium 02 · 242-97 45 · Brazil (011) 246-8149/246-8166 · Canada (514) 695-8225 · Finland (90) 80 17 044
France (1) 64 57 20 10 · Federal Republic of Germany (04106) 4055 · Great Britain (01) 954-2366 · Holland 03 402-39 994 · Hong Kong 5-487486 · Italy (02) 52 44 141
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Sweden (08) 711 27 30 · Switzerland (042) 65 11 61 · Taiwan (02) 713 9303 · USA (508) 481-7000 · Local representatives and service organisations world-wide

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